Exercise 2.2.7

Analyze the following equations graphically. In each case, sketch the vector field on the real line, find all the fixed points, classify their stability, and sketch the graph of $x(t)$ for different initial conditions. Then try for a few minutes to obtain the analytical solution for $x(t)$; if you get stuck, don't try for too long since in several cases it's impossible to solve the equation in closed form!

$$\dot{x} = e^x - \cos x$$

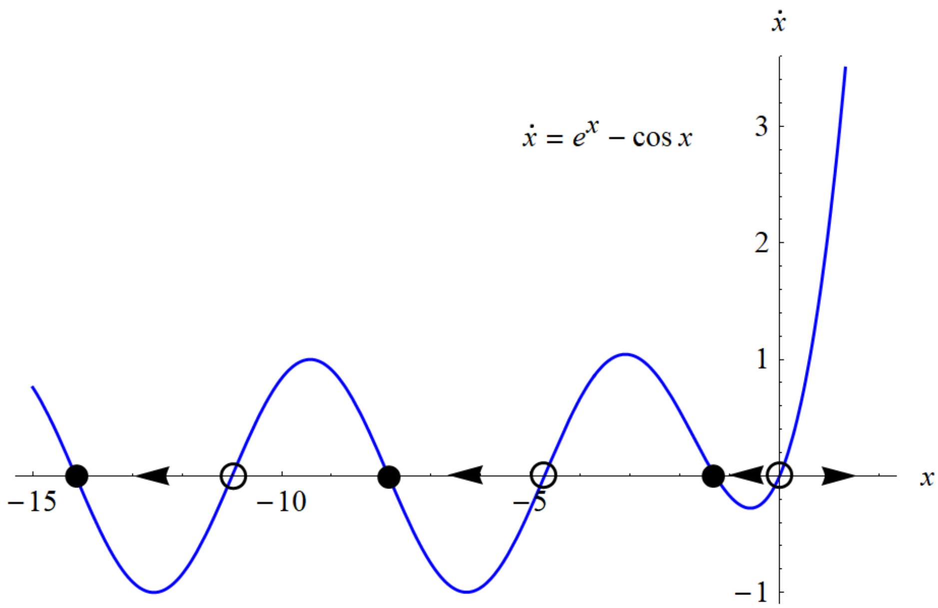
(Hint: Sketch the graphs of e^x and $\cos x$ on the same axes, and look for intersections. You won't be able to find the fixed points explicitly, but you can still find the qualitative behavior.)

Solution

The fixed points of the flow occur where $\dot{x} = 0$.

$$e^{x^*} - \cos x^* = 0$$

Plot the graph of $\dot{x}$ versus x in order to find them and determine whether each is stable or unstable.



When the function is negative the flow is to the left, and when the function is positive the flow is to the right. $x = 0$ is one fixed point, $x \approx -1.29270$ is another, and the rest are given roughly by

$$\cos x^* \approx 0$$

$$x^* \approx \frac{1}{2}(2n - 1)\pi, \quad n = 0, \pm 1, \pm 2, \ldots.$$

A qualitative sketch of x versus t is shown for various initial conditions.